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Entanglement of qubits via a nonlinear resonator

M Kurpas, J Dajka and E Zipper

Institute of Physics, University of Silesia, Ulica Uniwersytecka 4, 40-007 Katowice, Poland

E-mail: mkurpas@us.edu.pl

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Abstract
Coherent coupling of two qubits mediated by a nonlinear resonator is studied. It is shown that the amount of entanglement accessible in the evolution depends on both the strength of nonlinearity in the Hamiltonian of the resonator and on the initial preparation of the system. The created entanglement survives in the presence of decoherence.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Developments in quantum information science rely critically on entanglement–quantum correlations between two or more physical systems, e.g. between two qubits. There are various ways of creating entangled states of two qubits. The most natural is to place the qubits close to each other so that they can interact directly by local interactions, i.e. a mutual inductance or capacitance. There have been spectacular experiments performed for solid state qubits using this type of interaction [1–5]. However, the important step towards implementing quantum computation is a controllable coupling of qubits.

The entanglement of distant qubits can be reached by swapping of entanglement [6–8] or by some 'interaction-carrying' medium: a bus. In the following we discuss the latter case. One of the natural candidates for such a bus are photons that are highly coherent. The problem of interaction of qubits with a quantized monochromatic electromagnetic field in a quantum cavity has been discussed in several papers [9–13]. The field mediating the entanglement is usually described by a linear resonator [9–11, 14, 15]. However, it has been shown [11] that strong entanglement of qubits in a linear coupling regime is possible only for specific initial states. In this paper we want to check whether nonlinear radiation improves this situation.

Nonlinearity of a superconducting circuit resonator operating at microwave frequencies results in highly non-classical properties of the electromagnetic radiation generated by the resonator [16]. Such circuits coupled to qubits have recently been intensively studied [17] since they provide a natural measuring device [18, 19]. On the other hand, the problem of a nonlinear oscillator mediating the interaction between qubits is still open for theoretical investigations. A pair of qubits coupled via a driven Duffing oscillator has been studied in [20] with emphasis given to the (semi)classical limit of the oscillator. In some coupling schemes [21–23] an intermediate qubit with one or more Josephson junctions plays the role of a coupler. As the junction is strongly nonlinear, the coupler is treated as a nonlinear resonator.

The nonlinear character of coupling has been also broadly exploited in quantum optics: there are many proposals where nonlinear coupling effects play an essential role in generation of entanglement of photons [24–27] and in construction of quantum gates [28] or teleportation protocols [29].

In this paper we assume two qubits strongly coupled to a resonator and discuss coherent, non-local coupling between qubits via a linear and nonlinear resonator. The goal is to determine conditions which allow strong entanglement of the qubits with each other with the disentangled resonator. Thus a careful study of properties of the quantum bus is needed in order to fulfill this task. We investigate the influence of the strength of nonlinearity on coherent coupling of qubits and we show that it is highly nontrivial. On the basis of the obtained results we propose a scheme for a tunable entangling gate.

The results are applicable to entanglement through optical and microwave nonlinear radiation under the assumption that the qubit–field interaction is sufficiently weak to be described in the rotating wave approximation [30].

In the first part of the paper we assume that all effects of dissipation and decoherence are negligible over the studied timescales. We briefly discuss the effect of dissipation in section 7. To quantify the entanglement we calculate the negativity N [31] which is a natural measure of entanglement. Our discussion is general enough to not depend on the specific architecture. However, a candidate for possible implementation is the flux or charge qubit interacting with microwave radiation.

In section 2 we present the model of the system, in sections 3.1 and 3.2 we trace the evolution of the originally disentangled state with a *single excitation*. We consider two initial states which will be shown to result in qualitatively different behavior of the qubit–qubit entanglement: nonlinearity-assisted entanglement gain or entanglement suppression. In section 4 we investigate the system with *two* and *three excitations* and in section 5 we verify the results for the negativity by presenting the qubit–qubit density matrix. Finally, in section 6 we briefly study the influence of decoherence on qubit entanglement. Discussion and conclusions are given in section 7.

2. The model

The system under consideration consists of two qubits Q_i, $i = 1, 2$ and a monochromatic resonator R. We assume that both qubits are coupled to the resonator simultaneously. To simplify the notation we apply units such that $\hbar = 1$. In the absence of direct qubit–qubit interaction the Hamiltonian of such a system takes the form

$$H = H_{Q_1} + H_{Q_2} + H_R + H_{Q_1R} + H_{Q_2R}, \tag{1}$$

the qubit Hamiltonian is

$$H_{Q_i} = \frac{\Omega_i}{2}\sigma_z. \tag{2}$$

The interaction terms H_{Q_iR} read

$$H_{Q_iR} = -\frac{\gamma}{2}(a\sigma^+ + a^\dagger\sigma^-) \tag{3}$$

where $\sigma^+ = \sigma_x + i\sigma_y$, $\sigma^- = (\sigma^+)^\dagger$ and σ_z are the Pauli matrices and γ is the qubit–field coupling constant depending on the specific architecture. The resonator is described by a nonlinear bosonic oscillator of the Hamiltonian

$$H_R = \omega_R(a^\dagger a + \tfrac{1}{2}) + V_R, \tag{4}$$

where the term V_R describes the nonlinearity. We shall discuss two different forms of the nonlinearity. The first is a polynomial one

$$V_R^1 = \alpha(a^2 + (a^\dagger)^2) \tag{5}$$

which is known to be related to the celebrated squeezed states [32, 33]. The second one, of central interest for our considerations, is the cosine-like nonlinearity

$$V_R^2 = \alpha\cos(a + a^\dagger). \tag{6}$$

The cosine term introduces nonlinearities of all orders and in that sense the cosine is 'more nonlinear' than any polynomial. This type of nonlinearity is present, for example, in the Hamiltonian of the microwaves generated by a SQUID [1, 23, 34]. For such a circuit the parameter α can be changed by introducing into the resonator an adjustable weak link loop with two Josephson junctions. The Josephson tunneling energy can be controlled by an external flux ϕ_c; in this case $\alpha = 2E_J^0\cos(\pi\phi_c/\phi_0)$ [34, 35]. Varying ϕ_c we can change α between $2E_J^0$ and 0. A tunable anharmonic LC circuit is also important as it introduces non-uniform level spacing reducing leakage to higher states [21]. In the limit of $V_R = 0$ one arrives at a Hamiltonian of the Jaynes–Cummings type [36] for which exact solutions are known [11].

The state vector of the system at $t = 0$ is a tensor product of three constituent states

$$|\psi_{Q_1Q_2R}(t = 0)\rangle = |\psi_{Q_1}\rangle \otimes |\psi_{Q_2}\rangle \otimes |\psi_R\rangle. \tag{7}$$

This choice is certainly justified for weakly interacting systems. The unitary evolution of the initially factorizable state leads in general to the entangled tripartite state. By taking the trace over the R states one obtains the reduced density operator $\rho_{Q_1Q_2}$ which we then use to calculate the negativity [31]

$$N(\rho_{Q_1Q_2}) = \max\Big(0, -\sum_i \lambda_i\Big), \tag{8}$$

where λ_i are negative eigenvalues of the partially transposed [37] density matrix of the two qubits. N is 0 for separable states and reaches its maximal value $N = 1/2$ for maximally entangled states.

We look for the conditions under which the qubits get very strongly entangled.

3. Entanglement and nonlinearity

In the following we study the influence of the nonlinear term in the bosonic Hamiltonian (4) on the quantitative and qualitative properties of entanglement of the qubit–qubit system. We consider below only the cosine nonlinearity as we have found that the influence of the nonlinearity (5) is weaker and not qualitatively different. We compare the results with those obtained for the linear single mode resonator, which despite its simplicity is a natural reference system. For numerical analysis we truncated the photonic space to $M = 40$, for which $\mathrm{Tr}(|\psi_{Q_1Q_2R}(t)\rangle\langle\psi_{Q_1Q_2R}(t)| \simeq 1)$.

3.1. Entanglement gain

The first of the considered initial states is

$$|\psi_{Q_1Q_2R}(t = 0)\rangle = |eg0\rangle. \tag{9}$$

Here e and g stand for the excited and ground state, respectively, of one qubit. The state (9) describes the first qubit in the excited state, the second one in the ground state and there are no photons inside the resonator.

Let us start with an analysis of the system in the absence of the nonlinear term, i.e. $V_R = 0$. In this case one can perform exact analytical calculations of the wavefunction and trace the behavior of the system. The wavefunction of the system at the time t is [11]

$$|\psi_{Q_1Q_2R}(t)\rangle = \tfrac{1}{2}(1 + \cos(\tilde{\gamma}t))|eg0\rangle - \frac{1}{2}(1 - \cos(\tilde{\gamma}t))|ge0\rangle + \frac{i}{\sqrt{2}}\sin(\tilde{\gamma}t)|gg1\rangle, \tag{10}$$

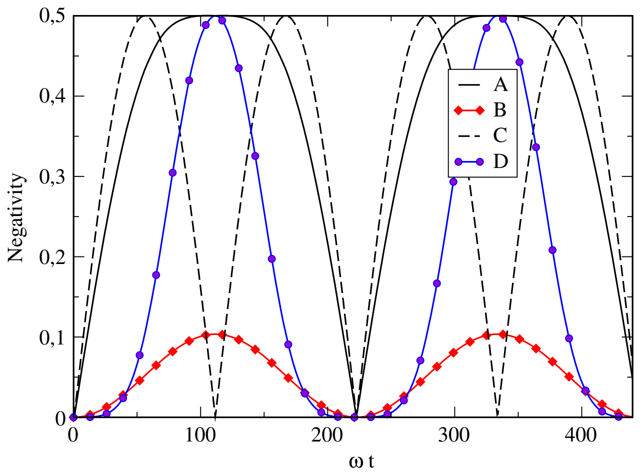

Figure 1. The QQ–R (line A) and QQ (line B) negativity as a function of dimensionless time ωt for the linear case and for the initial state $|eg0\rangle$. Lines C and D depict the QQ–R and QQ negativity for the initial state $|gg1\rangle$; $\Omega_i = \omega$, $\gamma = 0.01\omega$. Maximal qubit–qubit entanglement is possible when the resonator is not entangled with the qubits (lines C and D at $\omega t = 110$).

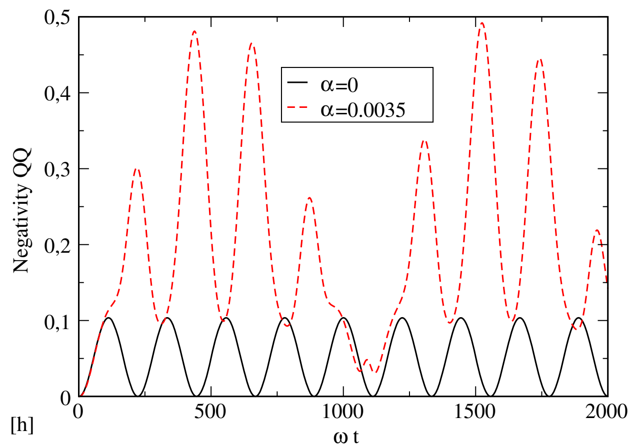

Figure 2. The unitary evolution of the qubit–qubit entanglement as a function of dimensionless time ωt and the nonlinearity coefficient α. The initial state $|eg0\rangle$. $\Omega_i = \omega$, $\gamma = 0.01\omega$.

where $\tilde{\gamma} = \sqrt{2}\gamma$. The time evolution goes in the subspace spanned by the states $|eg0\rangle, |ge0\rangle, |gg1\rangle$. For $\tilde{\gamma}t = n\pi$, $n = 1, 2, \ldots$. for which the R subsystem could be decoupled from the qubits; also the coefficient at the $|ge0\rangle$ term becomes zero and the whole system remains disentangled. This is shown in figure 1 as lines A (QQ–R entanglement) and B (QQ entanglement) at $\omega t \approx 220$. For other values of $\tilde{\gamma}t$ we are not able to obtain the entanglement of the qubits without the intrusion of the resonator states. This results in weak QQ entanglement with the limiting value $N_{QQ}^{max} \sim 0.1$. The inclusion of the nonlinear term ($V_R \neq 0$) allows us to go beyond this limit. Nonlinearity (even if very small) opens many new channels for entanglement and results in a significant increase of the entanglement of the qubits. It is shown in figure 2 that even for the relatively weak nonlinearity the negativity remains bounded by nothing but its natural limit, i.e. $N_{QQ}^{max} = 1/2$.

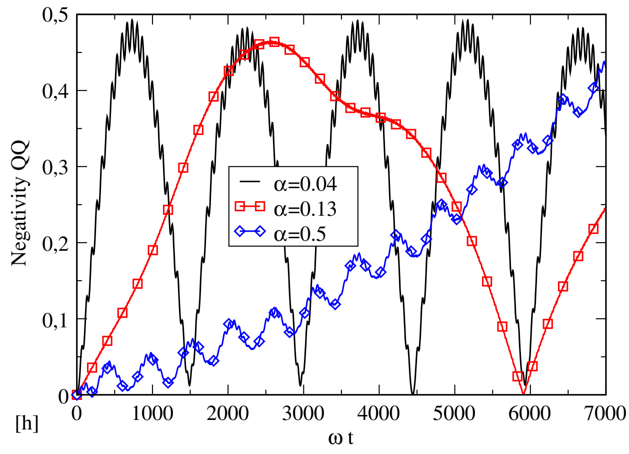

Figure 3. The qubit–qubit negativity for large values of the nonlinearity coefficient α. The initial state $|eg0\rangle$. $\Omega_i = \omega$, $\gamma = 0.01\omega$.

Increasing the strength of the nonlinearity (figure 3) the qubits get permanently entangled for a relatively long time which increases with increasing α. Thus we see that for the non-symmetric initial state, when the qubits start from opposite states, a nonlinear resonator can lead, in contrast to the linear one, to the emergence of strongly entangled QQ states remaining very weakly entangled with the resonator states. In other words we get coherent quantum state transfer between the qubits through the quantum bus.

3.2. Entanglement suppression

The situation looks different for the 'symmetric' initial state

$$|\psi_{Q_1Q_2R}(t=0)\rangle = |gg1\rangle, \tag{11}$$

when the excitation is placed in the resonator and both qubits are in the ground state. The analytical calculations for the linear resonator ($V_R = 0$) result in the wavefunction [11]

$$|\psi_{Q_1Q_2R}(t)\rangle = \frac{i}{\sqrt{2}}[\sin(\tilde{\gamma}t)|eg0\rangle + \sin(\tilde{\gamma}t)|ge0\rangle] + \cos(\tilde{\gamma}t)|gg1\rangle. \tag{12}$$

We see that at $\tilde{\gamma}t = \pi/2 + m\pi$, where m is an integer, we get maximally entangled qubit–qubit (Bell) state

$$|B_1\rangle = \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle). \tag{13}$$

It is important that the qubits can be in the maximally entangled state only, when their common state space is separable from the resonator space. This is shown in figure 1 (lines C—the QQ–R negativity and D—the QQ negativity) at $\omega t = 110$. As in the case of the initial state $|eg0\rangle$, the qubit–qubit negativity N_{QQ} reaches the value 0.1 (line D) when the QQ–R entanglement is maximal (figure 1 $\omega t = 50$ line C).

The inclusion of the nonlinear term spoils the symmetry (see equation (12)) and results in suppression of the qubit–qubit entanglement with the negativity limited by $N_{QQ}^{max} < 1/2$ depending on the amplitude α (see figure 4).

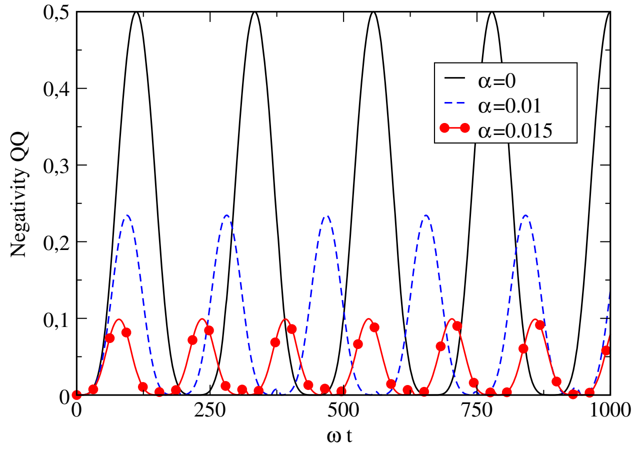

Figure 4. The influence of nonlinearity strength α of the resonator on the entanglement of the qubits for the initial state (11). $\Omega_i = \omega$, $\gamma = 0.01\omega$.

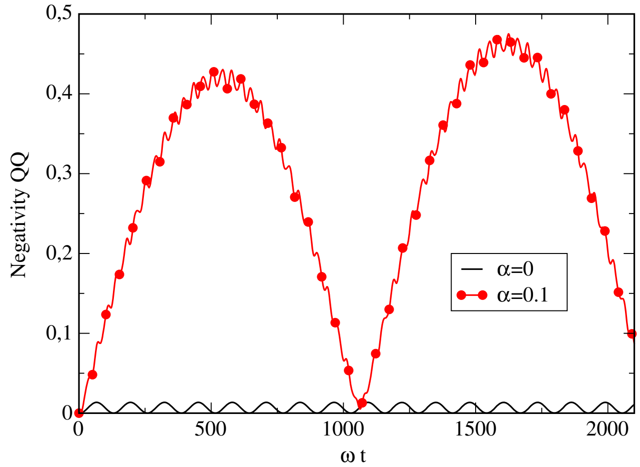

Figure 5. The influence of the nonlinearity strength α on the qubit–qubit entanglement for the initial state $|eg1\rangle$, $\Omega_i = \omega$, $\gamma = 0.01\omega$.

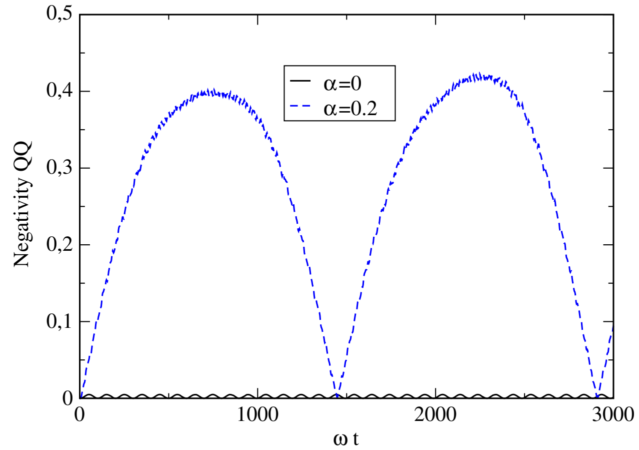

Figure 6. The influence of the nonlinearity strength α on the qubit–qubit entanglement for the initial state $|eg2\rangle$, $\Omega_i = \omega$, $\gamma = 0.01\omega$.

4. States with many excitations

Finally, let us proceed to a consideration of the evolution of the originally disentangled states with many excitations. In figures 5 and 6 we show the resulting qubit–qubit entanglement for the initial state with two and three excitations, respectively

$$|\psi_{Q_1Q_2R}(t=0)\rangle = |eg1\rangle \tag{14}$$

$$|\psi_{Q_1Q_2R}(t=0)\rangle = |eg2\rangle. \tag{15}$$

If the interaction between the qubits goes via a linear resonator ($\alpha = 0$) the negativity N_{QQ} (and hence the QQ entanglement) is very small (solid line). In other words, one is not able to get rid of the resonator degrees of freedom without leakage of the information about the state of the system. The situation changes if the interaction goes via the nonlinear resonator ($\alpha \neq 0$ lines in figures 5 and 6) and we obtain strong QQ entanglement. Thus in this case the influence of the nonlinearity of the 'quantum bus' is favorable for the creation of the entanglement of the qubits.

If we start from

$$|\psi_{Q_1Q_2R}(0)\rangle = |ee0\rangle \tag{16}$$

the interaction via the linear resonator does not entangle the qubits ($N_{QQ} = 0$). This is in agreement with exact analytical calculations [11]. We have also checked it by calculating the concurrence (not shown) which is another entanglement measure. Surprisingly, switching 'on' the nonlinearity does not improve the situation and the qubits remain disentangled.

For the initial state

$$|\psi(0)\rangle = |gg2\rangle \tag{17}$$

for the interaction via the linear resonator we get the QQ entanglement with $N_{QQ}^{max} \approx 0.18$. The presence of nonlinearity leads, in general, to a decrease of N_{QQ}^{max} but for some specific α ($\alpha \approx 0.7$) we have found a small increase of the negativity (not shown).

Summarizing, the influence of nonlinearity is favorable for the initial states with one qubit in the excited state. Then coupling via the quantum bus leads to strong entanglement and a coherent state transfer between the qubits.

5. Qubit–qubit density matrix

One of the best demonstrations of quantum entanglement is quantum state tomography which yields a density matrix of coherently coupled qubits [38]. Such experiments for qubits interacting via a resonant cavity are in progress [13]. As an example we present results of our calculations of the qubit–qubit density matrix for the case discussed in section 3.1.

In figure 7 we show the real part of the density matrix for qubits interacting via a linear resonator. The matrix elements are taken at $\omega t = 111$ for which $N_{QQ} = 0.104$ in figure 2. Looking at the state vector (10), this situation corresponds to an equal probability of finding the excitation in the resonator

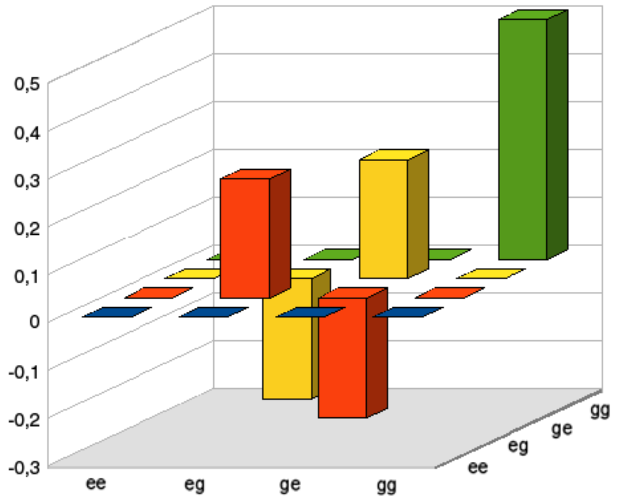

Figure 7. The real part of the qubit–qubit density matrix for the case of a linear resonator and the initial state (9). The matrix is taken at $\omega t = 111$ for which $N_{QQ} = 0.104$ (the solid line in figure 2). The imaginary elements of the matrix are of the order of 10^{-14}.

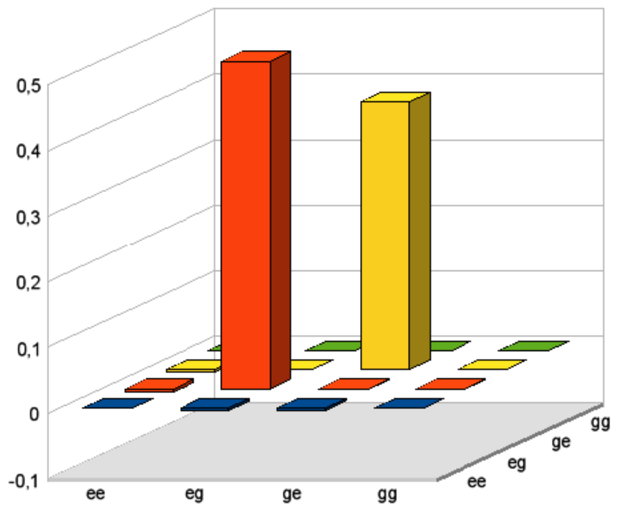

Figure 8. The real part of the qubit–qubit density matrix for the case of a nonlinear resonator ($\alpha = 0.0035$) and the initial state (9). The values are taken at $\omega t = 435$, $N_{QQ} = 0.492$ (the dashed line in figure 2).

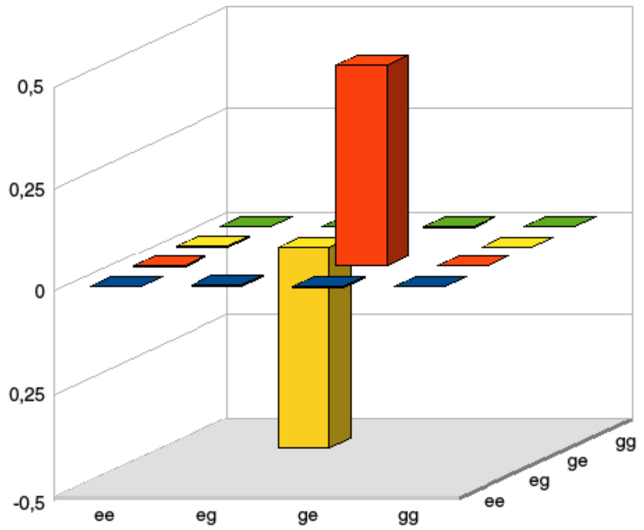

Figure 9. The imaginary part of the qubit–qubit density matrix for the case of a nonlinear resonator ($\alpha = 0.0035$) and the initial state (9). The values are taken at $\omega t = 435$, $N_{QQ} = 0.492$ (see the dashed line in figure 2).

or in one of the qubits. In figures 8 and 9 we present the real and imaginary part of the QQ density matrix for $\alpha = 0.0035$. We see that in this case the non-diagonal matrix elements, being the hallmark of entanglement, have much larger values than when the coupling goes via the linear resonator. The values correspond to the dashed line in figure 2 at $\omega t = 435$, $N_{QQ} = 0.492$.

6. The effect of decoherence

The design and construction of quantum devices is always affected by the influence of the environment. In the following we apply the commonly used Markovian approximation [30] and model the reduced dynamics of the QQR system in terms of a master equation generating complete positive dynamics [39]. We take into account the decoherence of both qubits and the resonator. Following [8, 9] we assume that the effect of the environment can be included in terms of three independent Lindblad terms:

$$\dot{\rho}(t) = [L_H - \tfrac{1}{2}L_{\rm D}]\rho(t) \quad (18)$$

where the 'conservative part' is given by

$$L_H(\cdot) = -{\rm i}[H, \cdot] \quad (19)$$

whereas the 'Lindblad dissipator'

$$L_{\rm D}(\cdot) = L_{\rm R}(\cdot) + \sum_{i=1} 2L_{{\rm Q}_i}(\cdot) \quad (20)$$

$$L_{\rm R}(\cdot) = A^\dagger A(\cdot) + (\cdot)A^\dagger A - 2A(\cdot)A^\dagger \quad (21)$$

$$L_{{\rm Q}_i}(\cdot) = \Sigma_i^\dagger \Sigma_i(\cdot) + (\cdot)\Sigma_i^\dagger \Sigma_i - 2\Sigma_i(\cdot)\Sigma_i^\dagger \quad (22)$$

is expressed in terms of creation and annihilation operators 'weighted' by suitable lifetimes $A = a/\sqrt{T_{\rm R}}$ and $\Sigma_i = \sigma^-/\sqrt{T_{{\rm Q}_i}}$, where $T_{\rm R} = 5 \times 10^{-5}$ s and $T_{{\rm Q}_i} = 10^{-5}$ s are the resonator and qubit decoherence times, respectively. It is shown in figure 10 that the effect of such a local damping is purely qualitative and the constructive role of the nonlinear term in the oscillator is not obscured provided that the relaxation times are sufficiently large.

7. Discussion and conclusions

In this paper we have considered the generation of entanglement in a system consisting of two qubits coupled by a linear or nonlinear resonator. The goal was to test the role of nonlinearity and to find conditions under which the qubits became strongly entangled. We have considered cosine-like nonlinearity where the discussed effects become most visible. We have found that the influence of the nonlinearity of the resonator, having its own dynamics, depends significantly on

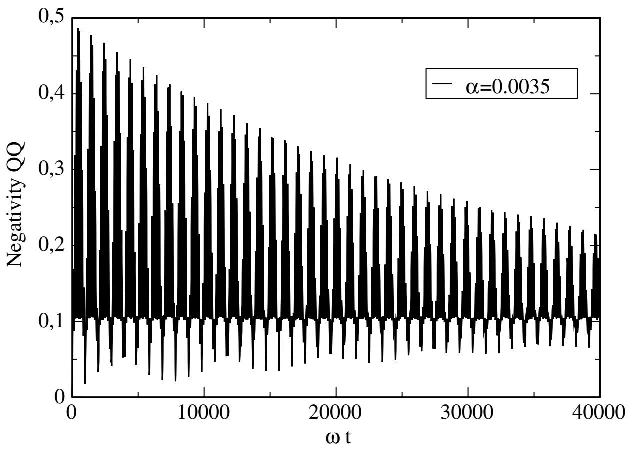

Figure 10. The influence of decoherence on the entanglement of the qubits for the initial state (9). $\Omega_i = \omega$, $\gamma = 0.01\omega$, $\alpha = 0.0035$, $T_R = 5\times10^{-5}$, $T_{Q_i} = 10^{-5}$ s.

the initial state of the investigated system. We have shown two qualitatively different behaviors. The first occurs in a system with non-symmetric initial states. The presence of nonlinearity results in strong enhancement of the entanglement almost up to its maximal value. For sufficiently large amplitude of the nonlinear term, the entanglement becomes (quasi) permanent and does not vanish for a long time. This constructive role of nonlinearity is certainly desired for applications. For symmetric initial states the role of nonlinearity is no longer constructive but rather results in the suppression of entanglement.

On the basis of these findings, we propose the following tunable coupling scheme: for the non-symmetric initial state the nonlinear term should be switched *on*, for the symmetric one this term should be *off* and the interaction goes via the linear resonator. Then in both cases we obtain the desired strong entanglement of qubits and the system acts as a quantum entangling gate. To our knowledge, such a scheme has not yet been proposed.

The main advantage of this gate is that the strong entanglement of qubits can be reached for many initial states, even for states with many excitations (with some exceptions). We have also shown that the coherent coupling of qubits survives in the presence of dissipation assuming decoherence times in agreement with recent experiments [12, 13].

Acknowledgments

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